OFDM & DMT in a nutshell
(Track #2: xDSL Physical layer)

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Outline

Goal of this tutorial

After the following 100 minutes you should be able to answer the following questions:

- Why multicarrier?
- Why *exactly* do we need a cyclic prefix?
- What is the difference between OFDM and DMT?
- Why is the peak-to-average power ratio a problem in OFDM/DMT?
- Why is synchronisation a challenge in OFDM/DMT?
1. Introduction and motivation: time-dispersive channel
2. Idea of multicarrier modulation
3. OFDM and DMT
4. Challenge No. 1: PAR
5. Challenge No. 2: Synchronisation
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Time dispersion in a wireless channel

Multipath propagation leads to dispersion of the receive signal in time
Time dispersion in a wireline channel

Different frequency-components experience different attenuation and different delay $\rightarrow$ dispersion ("smearing") in time
Discrete-time linear time-invariant (LTI) dispersive channel

\[ s(n) = \sum_{k} s(k) \delta(n - k) \]

\[ r(n) = \sum_{k} s(k) h(n - k) \]
A discrete-time, linear, time-invariant, causal, time-dispersive channel is described by its impulse response $h(n) \neq 0, n = 0, 1, \ldots, M$ of length $M + 1$

The channel performs linear convolution:

$$r(n) = \sum_{k} s(k) h(n - k)$$

Tapped delay-line model
Linear convolution, denoted by $\ast$, of a length-$N$ input sequence $s(n), n = n_0, \ldots, n_0 + N - 1$ and $h(n)$ results in an output sequence

$$r(n) = h(n) \ast s(n) = \sum_{k} h(k)s(n-k) =$$

$$= s(n) \ast h(n) = \sum_{k} s(k)h(n-k), \quad n = n_0, \ldots, n_0 + N + M - 1$$

of length $N + M$.

Since each input sample is “spread out” over $M = \lceil \tau_{\text{max}} / T_S \rceil$ additional samples, we refer to $M$ as the dispersion of the channel ($T_S$ is the sampling period)
Time-dispersion $\leftrightarrow$ frequency selectivity

**Impulse response**
- **Amplitude**
  - Short response in time
  - $\tau_{\text{max}}$

**Frequency response**
- **Magnitude**
  - Flat frequency response

**Amplitude**
- Long response in time
- $\tau_{\text{max}}$

**Magnitude**
- Selective frequency response

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Intro and motivation
Idea of multicarrier modulation
OFDM and DMT
PAR
Synchronisation
Communication: wideband versus narrowband

\[ T_{\text{sym}} \ll \tau_{\text{max}} \]  

\[ T_{\text{sym}} \gg \tau_{\text{max}} \]
Effect of dispersion on block transmission: ISI

- Length-$N$ symbol: $s(0), s(1), \ldots, s(N - 1)$
- Transmission of length-$N$ symbols over an LTI channel with dispersion $M$ (length $M + 1$) causes inter-symbol interference (ISI):
Dilemma: data rate and intersymbol interference

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<thead>
<tr>
<th>Example</th>
<th>System 1:</th>
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<tbody>
<tr>
<td></td>
<td>max. delay spread $\tau_{\text{max}} = 8 \mu s$</td>
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<tr>
<td></td>
<td>data rate $R = 100 \text{ kbit/s, BPSK}$</td>
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<td>$\rightarrow$ symbol duration $T_{\text{sym}} = ____$</td>
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<td>$\rightarrow$ ISI affects _____ symbol(s)</td>
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<tr>
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<td>max. delay spread $\tau_{\text{max}} = 8 \mu s$</td>
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<tr>
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<td>data rate $R = 10 \text{ Mbit/s, BPSK}$</td>
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<tr>
<td></td>
<td>$\rightarrow$ symbol duration $T_{\text{sym}} = ____$</td>
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<td>$\rightarrow$ ISI affects _____ symbol(s)</td>
</tr>
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### “Real-world” examples

#### Example

**GSM**
- Guess: max. delay spread \( \approx \) ___
- Guess: symbol duration \( \approx \) ___
- Compute: ISI affects ___ symbol(s)
- Conclusion: ISI is manageable

#### Example

**ADSL** (assume: data rate = 8 Mbit/s, uncoded 16QAM, single-carrier)
- Guess: max. impulse response duration \( \approx \) ___
- Compute: symbol duration = ___
- Compute: ISI affects ___ symbol(s)
- Conclusion: ISI is severe!
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Single-carrier transmission

- **Time domain**
  - Graph showing signal over time with labels $T_{sym}$ for $0$, $2T_{sym}$ for $2$, and $3T_{sym}$ for $3$.

- **Frequency domain**
  - Graph showing a rectangular region labeled $B$ surrounded by labels $T_{sym}$ for $0$, $2T_{sym}$ for $2$, and $3T_{sym}$ for $3$.

- **Constellation diagram**
  - Chart with points labeled as $\{x_0^{(i)}, x_0^{(q)}\}$, $\{x_1^{(i)}, x_1^{(q)}\}$, and $\{x_2^{(i)}, x_2^{(q)}\}$.
Idea: transmit several longer symbols simultaneously

\[ B \]

\[ 3T_{\text{sym}} = T_{\text{MC}} \]

**Time domain**

**Constellation diagram**

\[ \{ x_0^{(i)}, x_0^{(q)} \} \]

\[ \{ x_1^{(i)}, x_1^{(q)} \} \]

\[ \{ x_2^{(i)}, x_2^{(q)} \} \]
Idea (cont’d): $N$ narrowband channels

- Multicarrier-symbol duration $T_{MC} \gg$ delay spread $\tau_{max}$
- $N$ narrowband channels with bandwidth $\Delta f = B/N$ instead of one wideband channel with bandwidth $B$
- A multicarrier symbol in baseband is $N = \lceil T_{MC}/T_S \rceil$ samples long
- Subcarrier (tone) spacing $\Delta f = B/N = 1/T_{MC}$
- Each subchannel is quasi frequency-flat
Idea (cont’d): cyclic extension

- We make each transmit symbol at least $\tau_{\text{max}}$ seconds (or $L \geq M$ samples, where $M = \lceil \tau_{\text{max}} / T_S \rceil$) longer.
- At the receiver, we remove this extension, which eliminates the ISI.
- If $T_{MC} \gg \tau_{\text{max}}$, the loss is bearable.

![Diagram showing cyclic extension with blocks and symbols](image-url)
Checkpoint!

After the following 100 minutes you should be able to answer the following questions:

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Cyclic extension

- We extend the beginning of each multicarrier symbol by \( L \geq M \) samples.
- Since each transmit signal component extends over an integer number of periods, this extension corresponds to copying the last \( L \) samples to the beginning.
- The same holds for a linear combination of transmit signal components, i.e., for the complete transmit signal.
- Cyclic extension is a cheap operation (copying).

\[
\begin{align*}
\tilde{s}_2^{(q)}(n) &\quad \text{for transmit signal component} \\
\tilde{r}_2^{(q)}(n) &\quad \text{for complete transmit signal}
\end{align*}
\]
Why sinusoids?

- Small reasons: easy to generate (simple hardware), offer an intuitively appealing control over the spectrum, etc.

- BIG reason: infinitely-long sinusoids are eigen-functions of every linear time-invariant (LTI) system! Consider the LTI channel model:
  - Input: sine wave $\rightarrow$ output: transient ($\tau_{\text{max}} = MT_S$ seconds long) followed by shifted and scaled sine wave
  - The transient is eliminated by removing the cyclic extension at the receiver
  - The influence of the channel reduces to scaling and shifting in time (2 parameters = 1 complex coefficient per subcarrier)
  - We need the cyclic extension to produce the transient so that channel answers to our symbols as if they were infinitely long sinusoids
  - A cyclic extension of at least $\tau_{\text{max}}$ seconds ($L \geq M$) avoids not only ISI but also inter-carrier interference (ICI)
After the following 100 minutes you should be able to answer the following questions:

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**Choice of multicarrier symbol length** $T_{MC}$ — limits and design criteria

\[ \tau_{\text{max}} \ll T_{MC} \ll T_{\text{coh}} \]

- **Lower limit on:** $T_{MC} \gg \tau_{\text{max}}$ keeps loss due to cyclic extension low
- **Upper limit on:** the channel properties should not change during a symbol $\rightarrow T_{MC} \ll T_{\text{coh}}$ (otherwise our linear *time-invariant* channel model is invalid)
- The coherence time $T_{\text{coh}} = 1/B_{\text{dop}}$ is a measure for the duration the channel properties remain quasi constant
- Variation in time causes dispersion in frequency over $B_{\text{dop}}$ Hz
- Dispersion in frequency occurs for example due to motion: Doppler effect
When the fixed terminal (FT) transmits a signal with frequency \( f = c / \lambda \), the mobile terminal (MT) receives this signal at frequency \( f + \nu = f + \nu' / \lambda \), where \( \nu' \) is the relative velocity of the MT with respect to the FT and \( c \) is the propagation speed (for light \( c \approx 3 \cdot 10^8 \) m/s).

- Note that \( \nu' \) is a signed quantity

\[ \nu' = \nu_1 \cos \alpha_1 \]

\[ \nu_2 = -v_2 \cos \alpha_2 \]

- \( \nu \) is called the **Doppler shift** (example GSM: maximum Doppler shift for 100 km/h and 900 MHz: \( \nu = \nu' f / c \approx 83 \) Hz)
Choice of multicarrier symbol length $T_{MC}$ — limits and design criteria

Example

HiperLAN2 (pedestrian mobility: $v \leq 15$ km/h, $f = 5.2$ GHz, $B = 20$ MHz, $N = 64$):

- Guess: max. dispersion in time $\approx$ ____
- Compute: max. dispersion in frequency (due to motion): $T_{coh} =$ ____
- Compute: symbol duration $T_{MC} =$ ____
- Conclusion: $\tau_{max}$ $T_{MC}$ $T_{coh}$
  $\rightarrow$ HiperLAN2 should / should not work
The “O” of OFDM

- Experiment: observation of Fourier transforms of several sinusoids
- We observe that there are certain values of the carrier spacing which do not lead to inter-carrier interference.
- Carrier waveforms with such a spacing in frequency are mutually orthogonal.
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OFDM and DMT

- **OFDM**: Orthogonal Frequency Division Multiplexing
- **DMT**: Discrete Multi-Tone

DMT: **real-valued** baseband multiplex $s(n)$, OFDM: **complex-valued** baseband multiplex $s(n)$

DMT: **wireline** communications (lowpass channel), OFDM: **wireless** communications (passband channel)

DMT: channel **known** at the transmitter, OFDM: channel **unknown** at transmitter
OFDM subcarrier waveforms

- $v^{(i)}_k(m)$ (cosine signals, left column) and $v^{(q)}_k(m)$ (sine signals, right column); parameters: $N = 8$, $p = 32$, $f_c = 1/32$, $k = -3, \ldots, 4$
- There are $2N$ mutually orthogonal real-valued length-$N$ discrete-time sinusoidal waveforms with an integer number of periods
OFDM subcarrier waveforms

Oversampled (passband) waveforms:

\[
\begin{align*}
    u_k^{(i)}(m) &= \frac{1}{\sqrt{N}} \cos(2\pi \left( f_c + \frac{k}{Np} \right) m), \\
    u_k^{(q)}(m) &= -\frac{1}{\sqrt{N}} \sin(2\pi \left( f_c + \frac{k}{Np} \right) m),
\end{align*}
\]

\(m = -Lp, \ldots, Np - 1,\)

We write the passband transmit signal as

\[
\begin{align*}
    u(m) &= \sum_{k=-N/2+1}^{N/2} \left( x_{[k]}^{(i)} u_k^{(i)}(m) + x_{[k]}^{(q)} u_k^{(q)}(m) \right) \\
    &= \text{Re} \left\{ \frac{1}{\sqrt{N}} \sum_{k=-N/2+1}^{N/2} x_{[k]} e^{j2\pi \frac{k}{Np} m} e^{j2\pi f_c m} \right\},
\end{align*}
\]

where \([k]_N\) denotes the modulo-\(N\) operation applied to \(k\).
OFDM baseband transmit signal

Perfect interpolation by factor $p$ of $s(n)$ yields $s(m)$, where

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{j2\pi \frac{k}{N} n},$$

which is the IDFT of $x_k$

- Using the IDFT we can modulate $N$ subcarriers with only $N \log_2 N$ essential operations!
- The OFDM baseband transmit signal $s(n)$ is complex valued. I/Q modulation (sin/cos modulation) yields the real-valued passband transmit signal $u(m)$. 
DMT subcarrier waveforms

- There are $N$ mutually orthogonal real-valued length-$N$ discrete-time sinusoidal waveforms with an integer number of periods ($N = 8$ in this example).
- For $k \in \{ \lceil N/2 \rceil + 1, \ldots, N-1 \}$, the number of oscillations per $N$ samples decreases with increasing $k$.
- In fact, it is easy to verify that

  $$\cos\left(2\pi \frac{k}{N} n\right) = \cos\left(2\pi \frac{(N - k)}{N} n\right),$$
  $$\sin\left(2\pi \frac{k}{N} n\right) = -\sin\left(2\pi \frac{(N - k)}{N} n\right),$$
DMT transmit signal

Baseband waveforms:

\[ s_k^{(i)}(n) = \frac{1}{\sqrt{N}} \cos\left(2\pi \frac{k}{N} n\right), \quad k = 0, \ldots, \left\lfloor \frac{N}{2} \right\rfloor, \quad n = -L, \ldots, N-1 \]

\[ s_k^{(q)}(n) = -\frac{1}{\sqrt{N}} \sin\left(2\pi \frac{k}{N} n\right), \quad k = 1, \ldots, \left\lfloor \frac{N-1}{2} \right\rfloor, \]

The transmit signal is given by

\[ s(n) = 2 \sum_{k=0}^{N-1} \left( x_k^{(i)} s_k^{(i)}(n) + x_k^{(q)} s_k^{(q)}(n) \right) \]
We make the following choice for the transmit symbols

\[ x_k^{(i)} = x_{N-k}^{(i)} \quad \text{and} \quad x_k^{(q)} = -x_{N-k}^{(q)}, \]

which implies \( x_0^{(q)} = x_{N/2}^{(q)} = 0 \) and ensures Hermitian symmetry of the transmit symbol blocks. Consequently, the DMT transmit signal can be written as

\[ s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{j2\pi \frac{k}{N} n} \]

for \( n = -L, \ldots, N - 1 \)

- Using the IDFT we can again modulate \( N \) subcarriers with only \( N \log_2 N \) essential operations!
- The DMT transmit signal \( s(n) \) is real valued (due to our choice (1), which ensures Hermitian symmetry of the frequency-domain data) and can thus be transmitted in baseband.
Transmission using orthogonal waveforms

- We transmit a linear combination \( s(n) = \sum_k x_k s_k(n) \) of length-\( N \) transmit signal components \( s_k(n) \). The component No. \( k \) is scaled by the transmit symbol \( x_k \).

- Each transmit signal component \( s_k(n) \) is mapped onto its corresponding receive signal component \( \tilde{r}_k(n) \) by the channel.

- The channel output signal

\[
\tilde{r}(n) = h(n) \ast s(n) = \sum_k x_k (h(n) \ast s_k(n)) = \sum_k x_k \tilde{r}_k(n)
\]

is a linear combination of the receive signal components \( \tilde{r}(n) \).
Receive signal processing: correlation

How do we separate the transmitted signal components at the receiver? We use correlation, a measure of similarity of two length-$N$ signals $a(n)$ and $b(n)$:

$$\langle a, b \rangle = \sum_{n=0}^{N-1} a(n)b^*(n).$$

Among all sequences $b(n)$ with same energy as $a(n)$, the sequence $b(n) = a(n)$ yields the largest correlation $\langle a, b \rangle$!

We compute the correlation

$$y_k = \langle r, r_k \rangle = \sum_{n=0}^{N-1} \left( \sum_m x_m r_m(n) \right) r_k^*(n) = \sum_m x_m \langle r_m, r_k \rangle,$$

of the receive signal $r(n) = s(n) * h(n)$ with the receive signal components $r_k(n) = s_k(n) * h(n)$. 
Receive signal processing

- Through away the cyclic extension
- Correlate the receive signal with each transmit signal component:

\[ y_k = \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} r(n) \cos\left(2\pi \frac{k}{N} n\right) - j r(n) \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} \sin\left(2\pi \frac{k}{N} n\right) \]

\[ = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(n) e^{-j2\pi \frac{k}{N} n} r(n) \]

- All \(2N\) real-valued (\(N\) complex-valued) correlations can be computed with \(N \log_2 N\) essential operations using the DFT:

\[ y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(n) e^{-j2\pi \frac{k}{N} n}, \quad k = 0, \ldots, N - 1 \]
Block diagram of an OFDM system

\[ \{x_k\}_{k=0}^{N-1} \xrightarrow{\text{IDFT}} \text{OFDM modulator} \leftrightarrow \text{CP} \xrightarrow{\uparrow p} \text{RF modulator} \xrightarrow{\times e^{j2\pi f_c m}} \text{RF channel} \xrightarrow{\times h_{LP}(m)} \text{RF demodulator} \xrightarrow{\downarrow p} \text{OFDM demodulator} \xrightarrow{\text{DFT}} \{y_k\}_{k=0}^{N-1} \]
Block diagram of a DMT system

\[ \{ x_k \}_{k=0}^{N/2} \xrightarrow{\text{IDFT}} \text{CP} \xrightarrow{h(n)} \text{CP} \xrightarrow{\text{DMT}} \{ y_k \}_{k=0}^{N/2} \]
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Probably one of the biggest drawbacks of OFDM: the high peak-to-average power (PAR) ratio of the transmit signal multiplex

**Deterministic** measure for peakiness of a signal (or a realisation of a random process):

\[
\text{PAR} = \frac{\max_n |s(n)|^2}{\frac{1}{N} \sum_{n=0}^{N-1} |s(n)|^2}
\]

**Stochastic** measure for peakiness of a signal (or a random process):

\[
\Pr(\text{PAR} > \text{peaklevel})
\]
Deterministic quantification: PAR of some signals

- constant signal: $s^{(0)}(n) = 1$, PAR = 1 (0 dB), \((\frac{1}{N} \sum |s^{(0)}(n)|^2 = 1)\)
- $s^{(1)}(n)$: sinusoidal signal, PAR = 3 dB, max $|s^{(1)}(n)| = 1.41$; \((\frac{1}{N} \sum |s^{(1)}(n)|^2 = 1)\)
- $s^{(2)}(n)$: samples are realisations of uniform distribution, PAR = 4.7 dB, max $|s^{(2)}(n)| = 1.71$ \((\frac{1}{N} \sum |s^{(2)}(n)|^2 = 1)\)
- $s^{(3)}(n)$: samples are realisation of Gaussian distribution, PAR = 15.3 dB, max $|s^{(3)}(n)| = 5.78$, \((\frac{1}{N} \sum |s^{(3)}(n)|^2 = 1)\)
Stochastic quantification of PAR

- PAR level must be specified together with a probability
- Diagram does not reveal possibly present clip noise
Consequences of high PAR values

- At the receiver: clip noise limits performance

- At the transmitter: clipping power-amplifier violates PSD specs
Implications of the PAR challenge

- high PAR-value requires linearity of the transmitter chain over a wide amplitude range
- analogue components must be designed to deliver the peak power while sustaining linearity
- high PAR-values thus cause a large power consumption, which is a problem for
  - power-aware devices
  - devices that suffer from the heat they produce themselves
- amendments to keep the PAR low exist
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Synchronisation

- **Carrier frequency/phase synchronisation (OFDM)** ... critical!
  - The RF oscillators of transmitter (modulator) and receiver (demodulator) are running independently—thus, frequencies and/or phases may be different.
  - The receiver needs to tune its oscillator or compensate for the offset.

- **Symbol clock (or timing) synchronisation (DMT, OFDM)** ... critical!
  - The system clocks (sampling frequencies) of transmitter and receiver are generated independently—thus, they may be different.
  - The receiver needs to tune its sampling clock.

- **Frame synchronisation (DMT, OFDM)**
  - The receiver must find the frame boundaries (before windowing or removal of the cyclic extension).
Synchronisation

In general, synchronisation consists of two tasks:

- estimation of an appropriate parameter (frequency offset, phase offset, time offset)
- actual offset correction based on the estimate

Depending on the modulation type, we distinguish

- absolute modulation/encoding → coherent demodulation: requires recovery of the subcarriers
- differential modulation/encoding → non-coherent demodulation: subcarrier recovery is not necessary

Regarding the estimation task, we distinguish

- methods that are supported by deliberately inserted synchronisation-assisting signals (pilot signals, synchronisation symbols)
- methods that operate without this assistance (often referred to as “blind” estimation algorithms)
Carrier frequency/phase synchronisation

Assume a $\Delta F_c$ between the carrier frequency of transmitter and receiver:

Fourier transforms of the carriers

$f_1$  $f_2$  $f_3$  $f_4$  $f_5$  frequency
Consequence of $\Delta F_c \neq 0$:

- each subcarrier of a multicarrier symbol ends up on a frequency position that is shifted by $\Delta F_c$ Hz compared to the transmitter $\rightarrow$ orthogonality of the subcarriers is gone
- receive constellation rotates
- impact of a normalised frequency offset $\Delta_c = \Delta F_c / (F_s / N)$ (normalised by the subcarrier spacing) on the $l$th receive symbol is described by

\[
y_l \approx x_l \left( \text{sinc} \left( \Delta_c \right) e^{j\pi \Delta_c} \right) + \sum_{k \neq l} x_k \text{sinc} \left( k - l + \Delta_c \right) e^{j\pi (k-l+\Delta_c)}
\]

- ICI caused by carrier offset

In each subcarrier, interference from all the other subcarriers occurs!
Carrier frequency/phase synchronisation cont’d

- Regarding the carrier offset correction, we distinguish:
  - controlled oscillator; the oscillator of the down-converter (mixer) is adaptively tuned such that it corrects the carrier offset right away;
  - freely running oscillator followed by correction; the oscillator is not tunable, which simplifies its implementation; the resulting carrier offset is corrected by a multiplication of the receive signal multiplex with $e^{-j2\pi\Delta F_c/F_s n}$ before the DFT block;

- Parameter estimation in many practical implementations is based on correlation

- Most schemes operate in stages
  - coarse lock (“acquisition”)
  - fine tuning of the offset (“tracking”)

- Most robust methods, especially in fading environments, employ pilot symbols
Timing recovery

Assume a difference $\Delta F_s = F_s^{(\text{transmitter})} - F_s^{(\text{receiver})}$ between the sampling clock of the transmitter and the receiver:

Fourier transforms of the carriers

$f_1$ $f_2$ $f_3$ $f_4$ $f_5$
Timing recovery cont’d

Consequence of $\Delta F_s \neq 0$:

- frequency spacing of receive signal’s DFT is larger or smaller than the spacing of the transmit signal’s DFT $\rightarrow$ orthogonality is lost
- symbol window drifts
- impact of a normalised sampling clock offset $\Delta_s = \Delta F_s / F_s^{(receiver)}$ on the $l$th receive symbol is described by

$$y_l \approx x_l \frac{e^{j2\pi l\Delta_s} - 1}{j2\pi l\Delta_s} + \sum_{k \neq l} x_k \frac{e^{j2\pi k(1+\Delta_s)l} - 1}{j2\pi k(1+\Delta_s)l}$$

$$\Delta_s \ll x_l \left(1 - j\pi l\Delta_s\right) + \sum_{k \neq l} x_k \frac{k\Delta_s}{k-l}$$ (1)

- many terms contribute to the inter-carrier interference
- tolerable distortion for a subchannel depends on constellation size and desired bit error rate performance
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Thank you!